Why the poor will always outnumber the rich

R D Kingdon

November 2001

If one were to measure the distribution of wealth across a population in a free market economy, one would find that the poor always outnumber the rich.[†] Typically, the distribution may be approximated as a lognormal, but this choice of model is pragmatic, *i.e.* based on considerations of simplicity and goodness-of-fit, with the system being considered too complex for there to be an underlying explanatory mechanism.

The question of the distribution of wealth across a population is very similar to that of the size distribution of airborne particles or aerosols. Hinds[‡] states that 'There is no good reason why particle sizing data should approximate the lognormal distribution, but it has been found to apply to most single-source aerosols.' However, Rice[¶] provides just such a reason, which is worth quoting in full:

> 'The distribution of the sizes of grains of particulate matter is often found to be quite skewed, with a slowly decreasing right tail. A distribution called the lognormal is sometimes fit to such a distribution, and *X* is said to follow a lognormal distribution if $\log X$ has a normal distribution. The central limit theorem gives a theoretical rationale for the use of the lognormal distribution in some situations. Suppose that a particle of initial size y_0 is subject to repeated impacts, that on each impact a proportion, X_i, of the particle remains, and that the X_i are modeled as independent random variables having the same distribution. At the first impact, the size of the particle is $Y_1 = X_1 y_0$; after the second impact, the size is $Y_2 = X_2 X_1 y_0$; and after the *n*th impact, the size is
>
> $$Y_n = X_n X_{n-1} \dots X_2 X_1 y_0 .$$
>
> Then
>
> $$\log Y_n = \log y_0 + \sum_{i=1}^{n} \log X_i$$
>
> and the central limit theorem applies to $\log Y_n$.'

[†] The same might be said for the population of a centralised economy, where everyone is poor. The present analysis assumes the existence of a market mechanism, however.

[‡] W C Hinds, *Aerosol Technology: Properties, Behavior, and Measurement of Airborne Particles*, p.84.

[¶] J A Rice, *Mathematical Statistics and Data Analysis, Second Edition*, p.173.

In suggesting this mechanism, it seems that Rice was thinking of grains of particulate matter that are in close proximity, *e.g.* grains of sand on a beach. In this situation it is plausible that the size reduction of each particle on each impact is proportional to the size of the impacting grain; and this would be the physical justification for Rice's assumption of reduction factors which are 'independent random variables having the same distribution'. In the case of airborne particles or aerosols, however, these assumptions do not hold: firstly, aerosol particles are not in close proximity; and secondly, the collision of two aerosol particles is more likely to lead to their *agglomeration* than their fragmentation. Presumably, it is these considerations which lead experts such as Hinds to reject the simple 'collision' mechanism as an explanation for the lognormal particle size distribution. However, this conclusion appears to be mistaken, because Rice's argument can be adapted to the case of aerosols, as follows. Suppose that a particle of initial size y_0 is subject to repeated impacts from other particles, and that on each occasion the two particles stick together, forming a new, larger particle. To avoid double-counting, we adopt the convention that, on impact, the larger particle increases in size, while the smaller particle is removed from the system. It follows that the size of the surviving particle is given by $Y_1 = (1 + X_1)y_0$, where X_1 is a factor between 0 and 1 such that $X_1 y_0$ is the size of the smaller particle. Supposing that random collisions occur between particles of all sizes, then the X_i are independent random variables having the same distribution, and after the *n*th impact the size is given by

$$Y_n = (1 + X_n)(1 + X_{n-1})...(1 + X_2)(1 + X_1)y_0 .$$

Then

$$\log Y_n = \log y_0 + \sum_{i=1}^{n} \log(1 + X_i)$$

and the central limit theorem applies to $\log Y_n$. In principle, the overall population should be adjusted to account for the removal of the smaller particle on each collision. However, this removal occurs at random across the population distribution, and will not affect its shape. Simple computer simulations confirm that the proposed 'random agglomeration' mechanism indeed gives rise to the lognormal distribution.

In a free market, situations arise which are similar to the random agglomeration of particles. For example, firms can expand in size and market share by buying-out other companies, or by gradual (incremental) growth. These options are analogous to 'agglomeration' and 'accumulation', respectively.† Therefore, where market conditions favour buyouts (as tends to be the case), one might expect the resulting size distribution of companies to be lognormal. Where incremental growth is favoured, one might expect the resulting company size distribution to be gaussian. As for the question of the distribution of wealth across a population: many of the social and economic structures that are usually associated with a free market economy (*e.g.* marriage, inheritance, investment, performance-related pay) favour the 'agglomeration' of wealth rather than its 'accumulation'. Accordingly, the distribution of wealth will be lognormal, and the poor will always outnumber the rich.
So stop whinging and get on with it.

---

† 'Accumulation' is taken to mean growth at a rate that is independent of the original size, *i.e.* it corresponds to an additive *increment* rather than a multiplicative *factor*.